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Optical Radar Results and Meteoric Fragmentation

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# Optical Radar Results and Meteoric Fragmentation

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### Optical Radar Results and Meteoric Fragmentation

Recent optical radar experiments have indicated the existence of a scattering layer in the upper atmosphere at an altitude between 110-140 km. The echoes are tentatively interpreted according to a model of meteoric fragmentation and related to other experimental results. By progressive fragmentation in its flight through the atmosphere a meteoroid should show an enhancement of cross section responsible for those echoes. The atmosphere would be working as a "filter," and the average size distribution of micrometeorites would vary with the height.

## 1. Summary of data and considerations on radar cross section.

Some recent optical radar observations (Fiocco & Smullin, 1963) of echoes from the upper atmosphere have been reported. The results of numerous successive nights of observations indicate, among other features, weak, sporadic echoes at altitudes between 110 and 140 km; between 100 and 110 km a noticeable reduction of the returned signal is observed.

The experiments have been briefly described in the above reference.

A more complete analysis of the data and the techniques utilized, which are relatively new and still under development, will be presented at a later date. The results of present interest are, however, summarized in figure 1.

The diagram gives the observed radar cross section per unit volume,

averaged over 10 km range intervals and over 13 nights during which observations were possible, in the month from 17 July to 16 August 1963. The wavelength of radiation transmitted by the radar is 0.694 micron.

We are concerned at present with an interpretation of the echoes obtained at heights above 100 km, i.e. with the presence of maxima in the radar cross section above 110 km, and of minima between 100 and 110 The diagram is the result of averaging the successive observations of each single night (several hundred pulses), and then averaging again the results of the 13 successive nights by giving to each night the same weight in the average. A few remarks have to be made with regard to the accuracy of these determinations. Some measure of accuracy is provided by the amplitude of the r.m.s. background fluctuations, which is equivalent to 1.5  $\times$  10<sup>-13</sup> ( $\mathbb{Z}/100$ )<sup>2</sup> cm<sup>2</sup>/cm<sup>3</sup> ( $\mathbb{Z}$  = height in km). Because of slow recovery and nonlinearity of the photodetector, however, the sensitivity of the receiver decreases with decreasing height. The effect is noticeable at altitudes below 90 km: the cross sections, as given in the diagram, are in defect at those ranges. No correction has been introduced, since the present discussion is limited to the results of observations at higher levels. In passing, however, we note that the echoes observed between 80 and 90 km, at the mesopause, are indicative of cross sections much smaller than those that would be expected in noctilucent clouds (Witt, Hemenway and Soberman, 1963). The appearance of these echoes could be related to the same process responsible for the formation of the noctilucent clouds. It is not our aim, in this paper, to give a detailed interpretation of the phenomena occurring at this height.

These echoes have been found to be sporadic, although the observa-

tions were made during a period of high meteoric activity. For instance, if the results are averaged adding up all the data regardless of the time of occurrence, the height of the maximum above 110 km is reduced, and the difference in cross section between maximum and minimum levels is smaller by a factor of two. This effect results from the relative importance to the average of some nights where a larger number of observations than usual were made and when no echoes were received.

It seems reasonable to assume that these echoes are produced by small particles, meteoroids. A knowledge of their shape and of the ratio of the size to the wavelength is therefore essential in order to establish their individual cross section. For a particle much larger than the wavelength the radar cross section  $\sigma$  is proportional to the geometrical cross section and independent of wavelength; for dimensions much smaller than the wavelength  $\lambda$ , the scattering cross section decreases with a  $\lambda^{-4}$  dependence. In the indermediate region of sizes, large fluctuations occur. Values of the ratio  $\sigma/\pi \underline{a}^2$  for totally reflecting spheres are given by Van de Hulst (1957) as a function of  $x = 2\pi \underline{a}/\lambda$ . Suppose that a given volume Q is fragmented into N totally reflecting spheres of radius a, the density remaining constant. The total radar cross section of the N spheres, taken as independent scatterers, will be

$$N\sigma = Q \frac{3\pi}{2\lambda} \frac{1}{x} \left( \frac{\sigma}{\pi \underline{a}^2} \right) . \tag{1}$$

A plot of  $\frac{1}{x} = \frac{\sigma}{\pi a^2}$  is shown in figure 2. As the wavelength of the radar is 0.694 micron, a reduction in size corresponds to a large increase in total cross section until an optimum size  $(x \approx 1, \underline{a} \approx 0.11 \text{ micron})$  is

reached beyond which the cross section decreases very rapidly. Thus one may conceive that on its flight through the atmosphere a meteoroid undergoes progressive fragmentation in such a way that the collective radar cross section of the fragments increases until an average optimum size is reached and that further break-up, at lower heights, is responsible for a successive reduction of the observed cross section.

In the real case the particles will not behave as totally reflecting scatterers; notice, however, that the choice of spherical shape is unfavorable because it minimizes the geometrical cross section for a given mass and is probably far from reality, since published photographs of "fluffy" particles (Hemenway and Soberman, 1962) indicate very complex shapes (figure 3). For these filamentary shapes the radar cross section is much larger than the geometrical cross section.

Since we are interested in obtaining from the measure of the collective radar cross section an order of magnitude estimate of mass density and influx, but we do not have any measurement of individual radar cross sections, we shall assume that at some stage in the descent the meteoroid breaks up into fragments of optimum size for observation at the radar wavelength. We shall assume further that when the optimum size is reached, the average radar cross section  $\sigma$  is 10 times larger than the average geometrical cross section A.

The problem of maximizing radar cross section for a given mass is well known in radar technology. The cross section of wires of different lengths ("chaff") is treated for instance by van Vleck, Block and Hamer-

Witt, Hemenway and Soberman (1963) have collected particles from the mesopause. They found a large number of submicron particles (figure 4) of a size that is not expected to scatter efficiently at a wavelength  $\lambda = 0.69$  micron.

#### We note that

- a) the particles of optimum size for a wavelength  $\lambda$  = 0.69 micron cannot exist in elliptic orbit in the solar system because of radiation pressure;
- b) the size distribution found by Witt, Hemenway and Soberman is continuous up to a lower limit of approximately 0.025 micron radius, beyond which a real cut-off seems established;
- c) we can exclude that the particles of larger size found by Witt,

  Hemenway and Soberman exist in elliptic orbit in the solar

mesh (1947). For example, the ratio of the radar cross section (averaged over all directions) to the geometrical cross section for reflecting cylinders of half-wave length is  $\overline{\sigma}/A = 114$ , 217, 412 for ratios length to radius 2L/a = 225, 300, 450, respectively. For a sphere this ratio at best is 3.6. Extrapolating, for a cylinder of ratio 2L/a = 20, we obtain a ratio  $\overline{\sigma}/A = 18$ . If the material has a specific inductive capacity  $\varepsilon/\varepsilon_0 = 6$ ,  $(\sin \theta_2)$ , the scattering cross section suffers a reduction by a factor of 0.4. Thus the choice of  $\sigma/A = 10$  for a particle of optimum size may be considered a fair estimate. If that cylinder is broken into fifteen spheres of radius a, the collective cross section thus obtained is reduced by more than a factor of  $10^{-3}$ .

system;

d) nothing can be said, however, with regard to the effect of radiation pressure on small submicron particles (Harwit, 1963).

We cannot exclude that these can exist in elliptic orbits, thus inferring the possibility of gaps in the size distribution of interplanetary matter in elliptic orbit.\*

Because of these considerations we are naturally led to take the presence of small submicron particles as a strong element in favour of the hypothesis of fragmentation in the Earth's system. In the absence of fragmentation a gap should exist within the region of observed sizes.

A remark has to be made on the possible corpuscular pressure on a dust particle in the interplanetary space. The proton flux of the solar wind at 1 astronomical unit from the sun has been observed with the Mariner II. A few protons per cm<sup>3</sup> with a velocity of the order of 200 - 300 km/sec radially oriented from sun have been detected.

A computation of the momentum transfer to a dust particle requires an evaluation of the collision cross section, which is not an easy matter (Shapiro, 1962). However, at the earth distance from sun the radial component of the acceleration is of the same order of the radiation acceleration only if the collision cross section is two or three orders of magnitude greater than the geometrical cross section.

## 2. Fragmentation.

In his basic work on the theory of micrometeorites, Whipple (1950) expressed the conditions for a compact meteoroid to enter the atmosphere without melting. That optical and radar meteors break up, however, has been recognized for a few years (Jacchia, 1955; McCrosky, 1955; Öpik, 1956; Hawkins, 1963; Hawkins & Southworth, 1963). In fact, as Jacchia points out, ". . fragmentation is not a sporadic phenomenon, but rather the rule, and for faint meteors the classical concept of a single-body meteor must be replaced with that of a cluster of breaking fragments."

Fluffy particles or aggregates of very small crystals (0.02 to 0.1 micron radius) held together, perhaps, just by cohesive force have been recovered (Hemenway & Soberman, 1962). Crushing strengths of the order of 100 dynes/cm<sup>2</sup> or less are not inconceivable for meteoroids (Mc-Crosky, 1955; Whipple, 1961).

At present, a theory of fragmentation will meet with considerable difficulties because of the lack of information on the physical properties of these aggregates and their behavior under collision with atmospheric molecules or ions.

The possible role of thermal effects in reducing the crushing strength of the meteoroid should be mentioned. For aggregates of the type of interest the thermal conductivity is presumably very low, and the lag in heat transfer from different parts of the body may cause local thermal stresses (Whipple, 1950). Because of the irregular shape, the local stress resulting from aerodynamic pressure could also reach

higher values than the frontal pressure itself. The possibility of a build-up of an electric charge during the entering phase and its effect on the mechanism of fragmentation should be considered. Therefore, an investigation based on the purely mechanical aspect of the phenomenon may provide only a crude approach to the process of fragmentation.

Let us call A the average collision (geometric) cross section and m the mass of the meteoroid. As usual, we neglect the gravitational force and write the equation of motion for the high velocity phase of the flight through the atmosphere:

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}\mathbf{t}} = -\frac{\mathbf{D}}{2} \frac{\mathbf{A}}{\mathbf{m}} \rho(\mathbf{z}) \mathbf{v}^2. \tag{2}$$

D is the drag coefficient (that for free molecular flow is close to 2), and  $\rho(z)$  the atmospheric density at altitude z above sea level.

Let us assume for the moment that the ratio A/m remains constant during the flight until fragmentation occurs. The height at which the aerodynamic pressure reaches its maximum value is obtained from the equation

$$\frac{1}{\rho^2} \frac{\mathrm{d} \mathbf{v}}{\mathrm{d} t} = 2 \frac{\mathbf{A}}{\mathbf{m}} \mathbf{v} . \tag{3}$$

If  $Z_R$  is the zenith angle of incidence, and  $\boldsymbol{\ell}$  the arc length of the path (rectilinear) we have

$$\cos \mathbf{Z}_{\mathbf{R}} d \boldsymbol{l} = - d \mathbf{Z}, \tag{4}$$

and (3) becomes

$$\frac{1}{\rho H_{\rho}} = \frac{2A}{m \cos Z_{R}} . \tag{5}$$

This relation, where H is the density scale height, gives for each value of  $\frac{A}{m \cos Z_R}$  the height Z\* where the aerodynamic pressure reaches its maximum value. Note that Z\* does not depend on the entering velocity  $v_m$ .

Using the table of the U.S. Standard Atmosphere, 1962, we obtain table 1. In the first column we have the characteristic heights Z\*, in which we are interested; in the second column, the corresponding value of A/m cos  $Z_R$  in cm<sup>2</sup>/gram; in the third column, the ratio v\*/v<sub> $\infty$ </sub> of the velocity v\* at time of maximum pressure to the velocity at  $\infty$ .

In table 2, we have the values of  $\underline{\mathbf{a}}$  as a function of the density and of

$$\left(\frac{\mathbf{A}}{\mathbf{m}}\right)^{**} = \frac{1}{2} \left(\frac{\mathbf{A}}{\mathbf{m} \cos \mathbf{Z}_{\mathbf{R}}}\right)^{*}$$

Table 3 shows the value of the maximum aerodynamic pressure in  ${\rm dynes/cm}^2 \ as \ a \ function \ of \ altitude \ and \ of \ v_{_{\infty}}, \ for \ the \ different \ values \ of \\ \left(\frac{A}{m}\right)^{**} \ as \ in \ the \ previous \ tables.$ 

If we assume that the micrometeorite breaks up before or at the time that the aerodynamic pressure reaches its maximum value, we may state that

- (a) for a dust particle with a given A/m and for a given  $\mathbf{Z}_R$  there is a lower limit  $\mathbf{Z}_O$  for the altitude where it can break up;
- (b) if the dust particle has been detected at a height below  $\mathbf{Z}_{_{\mathrm{O}}}$ , it will not break up any more.

Thus a dust particle with a given A/m and  $Z_R$  may be released from a larger meteoroid at an altitude below its proper  $Z^*$ , but its lifetime

will be either zero or  $+\infty$ , because it will either break up immediately, being below Z\* with a higher velocity than it would have if it were coming from  $\infty$ , or it will not break up at all.

Having thus formulated these important conditions of purely mechanical character, we note that the possibility of progressive break-up caused by aerodynamic pressure seems to be critically dependent on how the ratio A/m and the crushing strength  $\tau$  of the meteoroid varies throughout the flight. It seems reasonable to assume that the crushing strength  $\tau$  increases for irregular porous bodies or for dustballs when the size decreases. The ratio A/m depends only on the product  $\underline{a}\delta$ . Because the density  $\delta$  can hardly go below 0.05 and cannot go higher than 7, while  $\underline{a}$  may go from 100 microns to 0.02 micron, it seems natural to assume that the ratio A/m is generally increasing as  $\underline{a}$  decreases.

If we adopt as a working hypothesis that A/m and  $\tau$  (crushing strength) increase with a decrease of  $\underline{a}$ , in order to explain the change in optical cross section, we have to suppose either (a) that particles of optimum size for the radar are orbiting around the Earth (and the question of their origin would be open to discussion)\* or coming from

The amount of dust released from the moon and falling on Earth as a consequence of meteoric impact (Whipple, 1961) depends critically on the value of some parameters related to the physical property of the impinging and ejected meteors as well as to the nature of the moon's surface (Gault, 1963). This matter seems to be still open to question, as it is an evaluation of the amount of dust captured from hyperbolic orbit

hyperbolic orbits in the solar system, and that they break into smaller sizes (practically invisible to radar) around 110 km, or (b) that particles of larger sizes (and smaller A/m) that can exist in elliptic orbits in the solar system undergo progressive fragmentations, spending a certain fraction of their flight in the typical optimum size and eventually disappearing, by the same mechanism. Typically, in this second situation, a meteoroid of 10 microns' radius or more, with a density between 0.1 and 0.5 \* and entering velocity of 30 km/sec will begin to fragment in sizes of the order of 0.1 - 0.5 micron at a height of 120 - 140 km (which is above the height Z\* previously defined) if it has a crushing strength of a few ten dynes/cm<sup>2</sup>; these fragments of density close to 1 will keep breaking up in flight downwards into smaller crystals typically of 0.02-0.05 micron at an altitude of 110 - 120 km. having reached then a speed of about 10 km/sec, if they have a crushing strength of the order of 100 dynes/cm<sup>2</sup>. To account for the minimum in the echoes between 100 and 110 km, it is necessary to consider at the

in elliptic orbit with respect to the Earth by means of a high drag process in the upper atmosphere. Fragmentation of incoming meteoroids with a  $\mathbf{Z}_{R}$  close to 90° may also feed fine dust into orbit around Earth.

The problem of motion of micrometeorites in the Earth-sun-moon environment has been investigated by Shapiro, Lautman, and Colombo (1963).

<sup>\*</sup> There are certainly some comets that have particles of extremely low density: the density for the comet Giacobini-Zinner, producing the great shower of 1946, has been evaluated to be of the order of 0.05 gm/cm<sup>3</sup> and even possibly less (Jacchia, Kopal and Millman, 1950).

same time the loss of cross section of an individual meteoroid of dimensions close to optimum when it is broken into n fragments and the increase in mass density resulting from slowing down because of atmospheric drag. If we consider that the terminal velocity of fragments of 0.05 diameter is about 50 m/sec at 100 km from an initial radial velocity of 5 km/sec, the decrease in cross section has to exceed two orders of magnitude, which is not inconceivable. For an initial vertical velocity of 5 km/sec at 110 km altitude, the vertical distance covered by a submicron particle before reaching terminal velocity is of the order of 10 km, and it is thus sufficient by itself to account for the presence of a minimum. The size distribution should be variable with height, and the atmosphere would be working as a "filter".

# 3. An estimate of influx and conclusions.

On the basis of the radar observations and of the present model of fragmentation an attempt can be made to estimate the influx on Earth of meteoric material that is undergoing fragmentation and is of a size suitable for observation. It is admittedly premature, in view of the very limited amount of observational data, to try to attach more than an order of magnitude significance to such an estimate.

From the size population found by Hemenway and Soberman it can be established that most of the total mass is obtained in sizes smaller than 0.1 micron of radius. According to our model all of this material is produced by fragmentation. Thus an estimate of the flux based on these particles of larger size that are observable by optical radar at

higher altitudes accounts for most of the mass obtained at lower levels. Some of the larger particles may well enter at lower speed in such a way that their contribution to the total flux will be small.

Since the experiments have been performed in a period of high meteoric activity and at night, a seasonal and diurnal rate factor affects the measurements. For the sake of obtaining an order of magnitude estimate we take the yearly average of the observed difference in cross section between the maximum above 110 and the minimum between 100 and 110 to be in the order of 10<sup>-13</sup> cm<sup>2</sup>/cm<sup>3</sup>. For particles of dimensions close to the optimum we shall assume an average radar cross section of the order of  $7 \times 10^{-9}$  cm<sup>2</sup>, an individual mass of the order of 2.5  $\times$  10<sup>-14</sup> gm; with a density of 1.5, and an average radial velocity of 5 km/sec. The present estimate depends on assuming a radial velocity consistent with an acceptable guess for the value of the crushing strength. Lower values of the crushing strength could bring a comparatively smaller influx. For instance, the existence of even a small electric potential on the meteoroid may be an important factor in causing the break-up (Singer, 1956; Opik. 1956). A total influx of the order of 6 × 10<sup>4</sup> tons a day on Earth is thus obtained: It is unnecessary to emphasize the great uncertainty attached to this estimate, in view of the scarcity of experimental data and the variety of assumptions. The number obtained is, however, in agreement with other published results. In figure 5, taken from a recent paper of G. S. Hawkins (1963), a star has been tentatively added to indicate the position of our estimate. This estimate is in fair agreement, within an

order of magnitude, with evaluations obtained from measurements of meteoric impact on satellites (Dubin & McCracken, 1962), although it disagrees with estimates based on interpretation of the measurement of the zodiacal light (Van de Hulst 1947, Whipple and Southworth personal communication). The hypothesis of fragmentation may account for the relatively low influxes obtained on the basis of Volz and Goody's (1962) twilight experiments (Carleton, 1962). We point out, however, that the estimates of the influx of extraterrestrial material are at present still uncertain (Whipple 1963).

Note at this point that the flux of extraterrestrial material as a function of the mass is critically dependent on the height where it is evaluated. With reference to figure 5 the dotted line passing through the star shows the effect of changing the individual mass of our particles but keeping the total mass constant, and may correspond to a changing of the height where the flux is measured. The use of different wavelengths, at the present time technically feasible, should provide a very valuable tool for establishing the size distribution of the dust at various heights.

In conclusion, the interpretation, based on progressive fragmentation of meteoroids, of the experimental results obtained with optical radar and with the recovery of micrometeorites by rocket, implies that most meteoroids are very weak in structure (dustballs), and that the radius of the smallest crystal that forms the structure of meteoroids lies between 0.1 and 0.01 micron.

We cannot (nor do we wish to) claim that this interpretation, which is, however, supported by other independent experimental results, is the only possible one. An exhaustive survey should consider, for instance, the role of winds and turbulence in the upper atmosphere as well as the suggested possibility of stimulated emissions by atmospheric constituents. In the lack of independent evidence and the difficulty of establishing quantitatively, their importance, we have at present preferred not to consider these concurrent processes.

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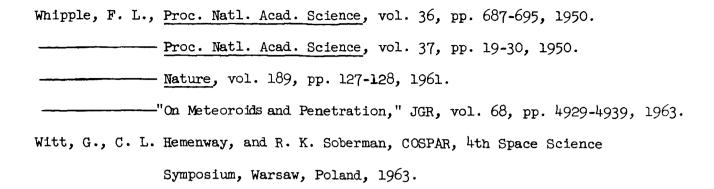
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#### REFERENCES

- Carleton, N. P., Journ. of Atmospheric Sciences, vol. 19, p. 424, 1962.
- Dubin, M., and C. W. McCracken, A.J., vol. 67, pp. 248-256, 1962.
- Fiocco, G., and L. D. Smullin, Nature, vol. 199, 1275-1276, 1963.
- Harwit, M., J. Geophys. Res., vol. 68, pp. 2171-2180, 1963.
- Hawkins, G. S., "The Meteor Population," <u>Proc. Conf. on Artificial Satellites</u>, NASA, NSF, Virginia Polytechnic Inst., Aug. 1963.
- Hawkins, G. S., and R. B. Southworth, "The Physical Characteristics of Meteors", NASA Research Report No. 2, (grant no. NAST-158), July 1963.
- Hemenway, C. L., and R. K. Soberman, A.J., vol. 67, pp. 256-266, 1962.
- Jacchia, L. G., Z. Kopal and D. M. Millman, Ap. J., vol. III, 104-133, 1950.
- Jacchia, L. G., Special Supplement (vol. II) to J. Atmos. Terr. Phys., p. 36, 1955.
- McCrosky, R. E., A.J., vol. 60, p. 170, 1955.
- Opik, E. J., <u>Irish Astron. Journ.</u>, vol. 4, pp. 84-135, 1956.
- Shapiro, I. I., "The Prediction of Satellite Orbits", in <u>Dynamics of Satellites</u>,

  JUTAM International Symp. Paris May 1962, Academic Press, N. Y. pp. 257312, 1963.
- Shapiro, I. I., D. Lautman, and G. Colombo, "Concentration of Dust Near the Earth", in print.
- Shoemaker, E. M., H. J. Moore, and D. E. Gault, NASA Technical Note D. 1767, 1963.
- Singer, S. F., 'Measurement of Interplanetary Dust', in Scientific Uses of Earth Satellites, Ann Arbor, 1956.
- van de Hulst, H. C., Ap. J., Vol. 105, pp. 671-488, 1947.
- van de Hulst, H. C., "Light Scattering from Small Particles," Wiley, New York, 1957.
- van Vleck, J. H., F. Block, and M. Hamermesh, J. Appl. Phys., vol. 18, p. 274, 1947.
- Volz, F. E., and R. M. Goody, Journ. of Atmospheric Sciences, vol. 19, p. 385, 1962.



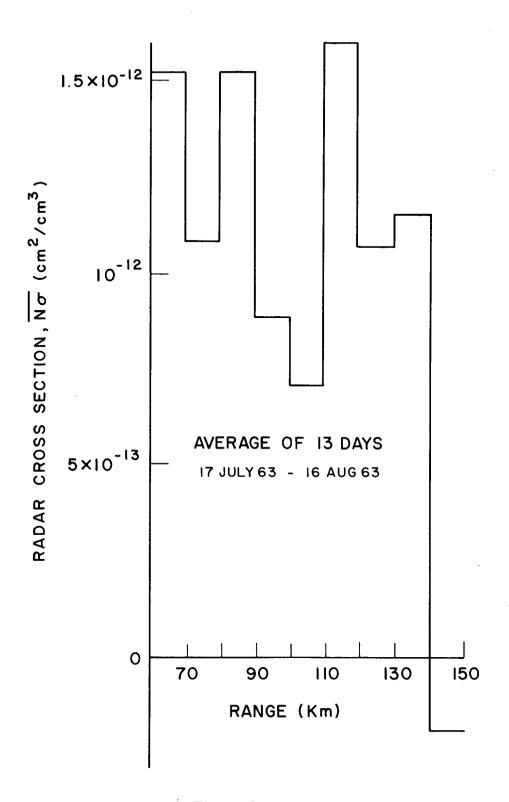


Figure 1

Average of observed radar cross sections per unit volume, versus range.

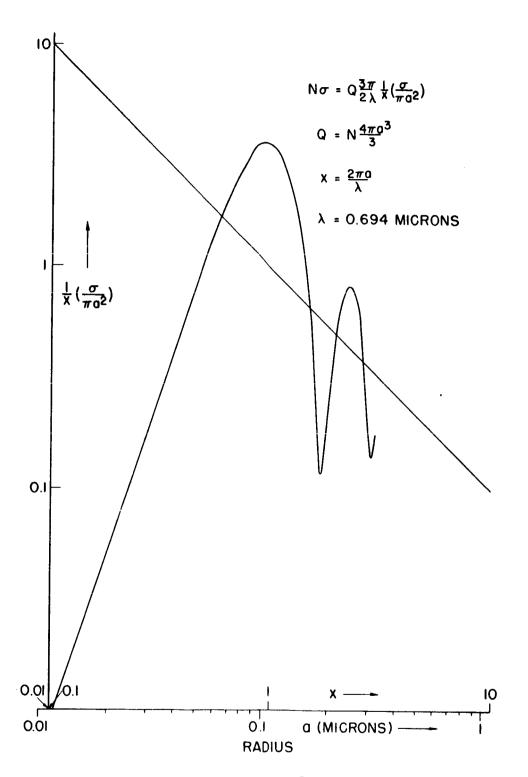


Figure 2

Radar cross section No of N totally reflecting spheres of equal radius  $\underline{a}$  and constant total volume Q.



Figure 3

A Fluffy Micrometeorite (Hemenway and Soberman, 1962)



Figure 4

Electron Micrograph of Cloud Collecting Surface
( Witt, Hemenway and Soberman, 1963)

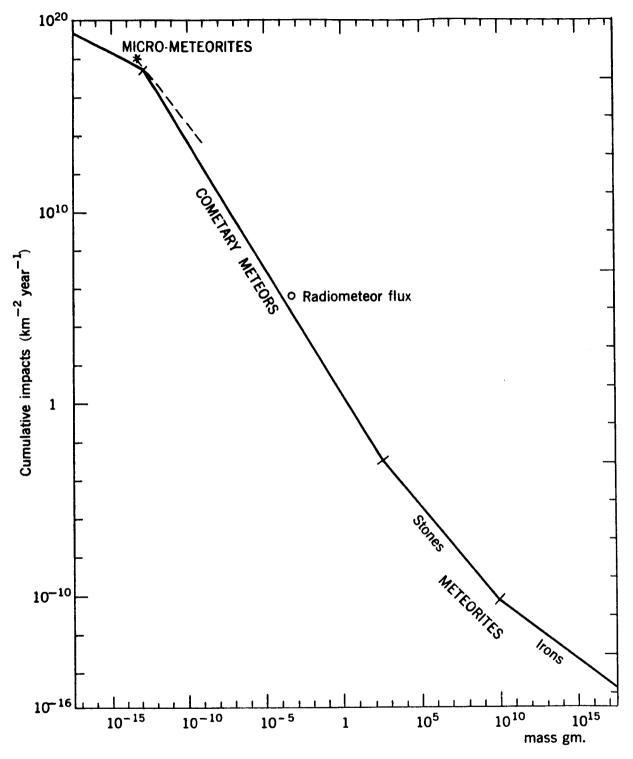


Figure 5

Flux of Extraterrestrial Objects

TABLE 1

Z* (km)	$\left(\frac{A}{m \cos Z_r}\right)^* \left(\frac{cm^2}{gram}\right)$	<u>v*</u> v∞
110	7.5 x 10 <sup>3</sup>	e <sup>-0.525</sup>
120	2.7 × 10 <sup>4</sup>	e-0.6
130	7.7 × 10 <sup>4</sup>	e <sup>-0.93</sup>
140	1.25 x 10 <sup>5</sup>	e-0.768

Value of  $\left(\frac{A}{m \cos Z_r}\right)^{**}$  and of the ratio  $\frac{v^*}{v_{\infty}}$  for which the maximum pressure is reached at an altitude  $Z^*$ .

TABLE 2

$\frac{\left(\frac{A}{m}\right)^{**}\left(\frac{cm^2}{grams}\right)}{\delta\left(grams/cm^3\right)}$	3.85 x 10 <sup>3</sup>	1.4 x 10 <sup>4</sup>	3.9 x 10 <sup>4</sup>	6.6 x 10 <sup>1</sup>
0.05	40	12	4	2.4
0.1	20	6	2.0	1.2
0.5	14	1.2	0.4	0.24
1	2	0.6	0.2	0.12
2	1	0.3	0.1	0.06
3	0.5	0.2	0.06	0.04

Radius  $\underline{a}$  in microns as a function of  $\delta$  and of  $\left(\frac{\underline{A}}{\underline{m}}\right)^{***}$ .

TABLE 3

	3.8 x 10 <sup>3</sup>	1.4 x 10 <sup>4</sup>	3.9 x 10 <sup>4</sup>	6.6 x 10 <sup>1</sup>
10	30	8	1	ı
20	120	32	4	4
30	270	72	9	9
40	480	120	16	9
50	750	200	25	25
60	1080	288	36	36

Maximum frontal pressure in dynes/cm² as a function of  $v_{\infty}$  and  $\left(\!\frac{A}{m}\!\right)^{\!*\!*\!*}$ 

#### NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory. First issued to ensure the immediate dissemination of data for satellite tracking, the Reports have continued to provide a rapid distribution of catalogues of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals.

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